Chapter 7

MyFinanceLab Solutions

Problem 7-1

(Related to Checkpoint 7.1) (Calculating rates of return) On December 24, 2007, the common stock of Google Inc. (GOOG) was trading for $700.73. One year later the shares sold for only $298.02. Google has never paid a common stock dividend. What rate of return would you have earned on your investment had you purchased the shares on December 24, 2007?

Step 1: Picture the problem

The rate of return you would have earned by buying Google's stock on December 24, 2007 and holding until December 24, 2008 is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Step 2: Decide on a solution strategy

The rate of return earned by holding Google's stock over the year period from December 24, 2007 to December 24, 2008 is calculated by the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price}}{\text{Beginning Price}} + \frac{\text{Cash Distribution (Dividends)}}{\text{Beginning Price}} - \frac{\text{Beginning Price}}{\text{Beginning Price}}
\]

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the holding period return.
Problem 7-1 (cont.)

Step 3: Solve

Substituting into the rate of return equation we get the following result:

\[
\text{Rate of Return, } r = \frac{\text{Ending Price} + \text{Cash Distribution} - \text{Beginning Price}}{\text{Beginning Price}} = \frac{298.02 + 0 - 700.73}{700.73} = -57.47\%
\]

The rate of return you would have earned is \(-57.47\%\).

Step 4: Analyze

Your investment in Google shares of stock yielded a negative 57.47\% rate of return. Compared to the rate of return earned on the general market — which was negative by only about half as much for the year — you probably aren't very happy with this return.
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Problem 7-2

(related to Checkpoint 7.1) (Calculating rates of return) The S&P stock index represents a portfolio comprised of 500 large publicly traded companies. On December 24, 2007, the index had a value of 1,410 and on December 24, 2008, the index was approximately 890. If the average dividend paid on the stocks in the index is approximately 4.0% of the value of the index at the beginning of the year, what is the rate of return earned on the S&P index? What is your assessment of the relative riskiness of investing in a single stock such as Google compared to investing in the S&P index (recall from Chapter 2 that you can purchase mutual funds that mimic the returns of the index)?

Step 1: Picture the problem

The rate of return you would have earned by buying the S&P 500 index on December 24, 2007 and holding until December 24, 2008 is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Keep in mind, a cash dividend of 4.0% of the value of the index at the beginning of the year adds to your investment return.

Step 2: Decide on a solution strategy

The rate of return earned by holding the S&P 500 index over the year period from December 24, 2007 to December 24, 2008 is calculated by the following equation:
Problem 7-2 (cont.)

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}}
\]

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the \textit{holding period return}.

**Step 3: Solve**

Substituting into the rate of return equation, we get the following:

Cash Dividend = 1410 \times 0.040 = $56.40

\[
\text{Rate of Return, } r = \frac{\text{Ending Price} + \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}} = \frac{890 + 56.40 - 1410}{1410} = -32.88\%
\]

Investing in a single stock is riskier than investing in the S&P index.

**Step 4: Analyze**

The investment in the S&P 500 index yielded a negative 32.88% rate of return.

In general, investing in a single stock is riskier than investing in the S&P index because indexes are similar to mutual funds and mutual funds have less risk than single stocks because of diversification.
Problem 7-3

(Related to Checkpoint 7.1) (Calculating rates of return) The common stock of Placo Enterprises had a market price of $12.00 on the day you purchased it just one year ago. During the past year the stock had paid a dividend of $1.00 and closed at a price of $14.00. What rate of return did you earn on your investment in Placo’s stock?

Step 1: Picture the problem

The rate of return you would have earned by buying Placo Enterprises stock a year ago and holding it for one year is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Keep in mind, a cash dividend of $1.00 during the year adds to your investment return.

Step 2: Decide on a solution strategy

The rate of return earned by holding Placo Enterprises stock for the one-year period is calculated by the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution} - \text{Beginning Price}}{\text{Beginning Price}}
\]

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the holding period return.
Problem 7-3 (cont.)

Step 3: Solve

Substituting into the rate of return equation we get the following result:

\[
\text{Rate of Return, } r = \frac{\text{Ending Price} + \text{Cash Distribution} - \text{Beginning Price}}{\text{Beginning Price}} = \frac{$14.00 + $1.00 - $12.00}{$12.00} = 25.00\%
\]

Step 4: Analyze

Your investment in Placo Enterprises stock yielded a positive 25.00% rate of return.
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Problem 7-4

(Related to Checkpoint 7.1) (Calculating rates of return) Blaxo Balloons manufactures and distributes birthday balloons. At the beginning of the year Blaxo’s common stock was selling for $20.00 but by year end it was only $18.00. If the firm paid a total cash dividend of $2.00 during the year, what rate of return would you have earned if you had purchased the stock exactly one year ago? What would your rate of return have been if the firm had paid no cash dividend?

Find the rate of return with the dividend included.

Step 1: Picture the problem

The rate of return you would have earned by buying Blaxo Balloons stock a year ago and holding it for one year is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Keep in mind, a cash dividend of $2.00 during the year adds to your investment return.

Step 2: Decide on a solution strategy

The rate of return earned by holding Blaxo Balloons stock for the one-year period is calculated by the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution (Dividends)}}{\text{Beginning Price}} - \frac{\text{Beginning Price}}{\text{Beginning Price}}
\]
Problem 7-4 (cont.)

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the holding period return.

Step 3: Solve

Substituting into the rate of return equation we get the following result:

\[
\text{Rate of Return, } r = \frac{\text{Ending Price} + \frac{\text{Cash Distribution}}{\text{(Dividends)}} - \frac{\text{Beginning Price}}{\text{Price}}}{\text{Beginning Price}} = \frac{\$18.00 + \frac{\$2.00}{\$20.00} - \frac{\$20.00}{\$20.00}}{\$20.00} = 0.00\%
\]

Step 4: Analyze

Your investment in Blaxo Balloons stock with the dividend paid yielded a rate of return of 0.00%.

Find the rate of return without including the dividend.

Step 1: Picture the problem

The rate of return you would have earned by buying Blaxo Balloons stock a year ago and holding it for one year is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.
Problem 7-4 (cont.)

Keep in mind, no cash dividend is paid in this case.

**Step 2: Decide on a solution strategy**

The rate of return earned by holding Blaxo Balloons stock for the one-year period is calculated by the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution} - \text{Beginning Price}}{\text{Beginning Price}}
\]

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the *holding period return*.

**Step 3: Solve**

Substituting into the rate of return equation we get the following result:
Problem 7-4 (cont.)

\[
\text{Rate of Return, } r = \frac{\text{Ending Price} + \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}} = \frac{\$18.00 + $0 - $20.00}{\$20.00} = -10.00\%
\]

**Step 4: Analyze**

Your investment in Blaxo Balloons stock without the dividend paid yielded a rate of return of $-10.00\%$. 
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Problem 7-5

(Computing rates of return) From the following price data, compute the annual rates of return for Asman and Salinas.

<table>
<thead>
<tr>
<th>Time</th>
<th>Asman</th>
<th>Salinas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
<td>$30</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>35</td>
</tr>
</tbody>
</table>

How would you interpret the meaning of the annual rates of return?

Step 1: Picture the problem

The rate of return you would have earned over a year period investing in Asman is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Step 2: Decide on a solution strategy

The rate of return earned by holding Asman stock for a one-year period is calculated by the following equation:
Problem 7-5 (cont.)

\[
\text{Rate of Return} = \frac{\text{Cash Return} + \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}}
\]

Note that the rate of return for holding an investment for a specific period of time is sometimes referred to as the *holding period return*.

**Step 3: Solve**

Substituting into the rate of return equation we get the following result for Asman:

\[
\text{Rate of Return, } r_{\text{end of time 2}} = \frac{\text{Price at time 2} + \text{Cash Distribution (Dividends)} - \text{Price at time 1}}{\text{Price at time 1}} = \frac{$12 + $0 - $10}{$10} = 20.00\%
\]

\[
\text{Rate of Return, } r_{\text{end of time 3}} = \frac{\text{Price at time 3} + \text{Cash Distribution (Dividends)} - \text{Price at time 2}}{\text{Price at time 2}} = \frac{$11 + $0 - $12}{$12} = -8.33\%
\]

\[
\text{Rate of Return, } r_{\text{end of time 4}} = \frac{\text{Price at time 4} + \text{Cash Distribution (Dividends)} - \text{Price at time 3}}{\text{Price at time 3}} = \frac{$13 + $0 - $11}{$11} = 18.18\%
\]
Step 4: Analyze

Your investment in Asman stock yielded an annual rate of return between time 1 and time 2 of 20.00%, between time 2 and time 3 of \(-8.33\)% , and between time 3 and time 4 of 18.18%.

Step 1: Picture the problem

The rate of return you would have earned over a year period investing in Salinas is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.

Step 2: Decide on a solution strategy

The rate of return earned by holding Salinas stock for a one-year period is calculated by the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}}
\]
Problem 7-5 (cont.)

**Step 3: Solve**

Substituting into the rate of return equation we get the following result for Salinas:

\[
\text{Rate of Return, } r = \frac{\text{Price at time 2} + \frac{\text{Cash Distribution (Dividends)}}{\text{Price at time 1}} - \frac{\text{Price at time 1}}{30}}{\text{Price at time 1}} = \frac{28 + 0 - 30}{30} = -6.67\%
\]

\[
\text{Rate of Return, } r = \frac{\text{Price at time 3} + \frac{\text{Cash Distribution (Dividends)}}{\text{Price at time 2}} - \frac{\text{Price at time 2}}{28}}{\text{Price at time 2}} = \frac{32 + 0 - 28}{28} = 14.29\%
\]

\[
\text{Rate of Return, } r = \frac{\text{Price at time 4} + \frac{\text{Cash Distribution (Dividends)}}{\text{Price at time 3}} - \frac{\text{Price at time 3}}{32}}{\text{Price at time 3}} = \frac{35 + 0 - 32}{32} = 9.38\%
\]

**Step 4: Analyze**

Your investment in Salinas stock yielded an annual rate of return between time 1 and time 2 of \(-6.67\%\), between time 2 and time 3 of 14.29\% and between time 3 and time 4 of 9.38\%.

How would you interpret the meaning of the annual rates of return?

The annual rate of return when there are no dividends paid is the price at the end of one period less the price at the beginning of the period divided by the price
Problem 7-5 (cont.)

at the beginning of the period. These returns can greatly vary from one period to the next.
Problem 7-6

(Related to Checkpoint 7.1) (Expected rate of return and risk) B. J. Gautney Enterprises is evaluating a security. One-year Treasury bills are currently paying 2.9% and have a standard deviation of 1.81%. Calculate the investment's expected return and its standard deviation. Should B. J. Gautney Enterprises invest in this security?

<table>
<thead>
<tr>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>-3 %</td>
</tr>
<tr>
<td>0.30</td>
<td>2 %</td>
</tr>
<tr>
<td>0.40</td>
<td>4 %</td>
</tr>
<tr>
<td>0.15</td>
<td>6 %</td>
</tr>
</tbody>
</table>

Step 1: Picture the problem

The distribution of possible rates of return for the investment along with the probabilities of each can be depicted in a probability distribution as follows:

The probabilities of each of the potential rates of return are read off the vertical axis and the returns are found on the horizontal axis.

Step 2: Decide on a solution strategy

We use the expected value of the rate of return to measure B.J. Gautney Enterprises' expected return from the investment and the standard deviation to evaluate its risk. We can use the equations below for these tasks.
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Problem 7-6 (cont.)

\[
\begin{align*}
\text{Expected Rate of Return} & = \left( \text{Rate of Return }1 \times \text{Probability }1 \right) + \left( \text{Rate of Return }2 \times \text{Probability }2 \right) + \ldots + \left( \text{Rate of Return }n \times \text{Probability }n \right) \\
\left[ E(r) \right] & = \left( r_1 \times P_{b1} \right) + \left( r_2 \times P_{b2} \right) + \ldots + \left( r_n \times P_{bn} \right) \\
\text{Variance in Rates of Return} & = \left[ \frac{\text{Rate of return for State 1} - \text{Expected Rate of Return}}{r_1} \times \text{Probability }1 \right]^2 + \left[ \frac{\text{Rate of return for State 2} - \text{Expected Rate of Return}}{r_2} \times \text{Probability }2 \right]^2 + \ldots + \left[ \frac{\text{Rate of return for State }n - \text{Expected Rate of Return}}{r_n} \times \text{Probability }n \right]^2 \\
\left( \sigma^2 \right) & = \left( \frac{E(r) - 1}{P_{b1}} \right)^2 + \left( \frac{E(r) - 2}{P_{b2}} \right)^2 + \ldots + \left( \frac{E(r) - n}{P_{bn}} \right)^2 \\
\end{align*}
\]

Step 3: Solve

(a) Calculating the expected return. We use the expected return equation given to calculate the expected rate of return for the investments as follows:

\[
E(r) = r_1P_{b1} + r_2P_{b2} + \ldots + r_nP_{bn}
\]

\[
E(r) = (-3\% \times 0.15) + (2\% \times 0.30) + (4\% \times 0.40) + (6\% \times 0.15) = 2.65\%
\]

(b) Calculating the standard deviation.

Next, we calculate the standard deviation using the equation given as follows:
Problem 7-6 (cont.)

\[
\sigma = \sqrt{([r_1 - E(r)]^2 P_b_1) + ([r_2 - E(r)]^2 P_b_2) + \ldots + ([r_n - E(r)]^2 P_b_n)}
\]

\[
\sigma = \sqrt{[(-3\% - 2.65\%)^2 (0.15) + (2\% - 2.65\%)^2 (0.30) + (4\% - 2.65\%)^2 (0.40) + (6\% - 2.65\%)^2 (0.15)]}
\]

\[
\sigma = \sqrt{7.33\%} = 2.71\%
\]

Step 4: Analyze

The expected return for the investment is 2.65\%; however, since there is a 15\% chance that the actual return may be 6\% and a 15\% chance that the actual return may be −3\%, it is obvious that this is a risky investment. In this example, the standard deviation, which is a measure of the average or expected dispersion of the investment returns, is equal to 2.71\%.

B. J. Gautney Enterprises should not invest in this investment because the return is lower than the Treasury and the level of risk higher than the Treasury.
Common Stock A | Common Stock B
---|---
Probability | Return | Probability | Return
0.30 | 11% | 0.20 | -5%
0.40 | 15% | 0.30 | 6%
0.30 | 19% | 0.30 | 14%

**Step 1: Picture the problem**

The distribution of possible rates of return for the investment in common stock A along with the probabilities of each can be depicted in a probability distribution as follows:

![Probability Distribution](image)

The probabilities of each of the potential rates of return are read off the vertical axis and the returns are found on the horizontal axis.

**Step 2: Decide on a solution strategy**

We use the expected value of the rate of return to measure expected return from the investment and the standard deviation to evaluate its risk. We can use the equations below for these tasks.
Problem 7-7 (cont.)

Expected Rate of Return

\[
[E(r)] = \left( \frac{\text{Rate of Return 1} \times \text{Probability of Return 1}}{(r_1) \times (Pb_1)} \right) + \left( \frac{\text{Rate of Return 2} \times \text{Probability of Return 2}}{(r_2) \times (Pb_2)} \right) + \ldots + \left( \frac{\text{Rate of Return } n \times \text{Probability of Return } n}{(r_n) \times (Pb_n)} \right)
\]

Variance in Rates of Return

\[
(\sigma^2) = \left[ \left( \frac{\text{Rate of return for State 1} - \text{Expected Rate of Return}}{(r_1) - E(r)} \right)^2 \times \text{Probability of State 1} \right] + \left[ \left( \frac{\text{Rate of return for State 2} - \text{Expected Rate of Return}}{(r_2) - E(r)} \right)^2 \times \text{Probability of State 2} \right] + \ldots + \left[ \left( \frac{\text{Rate of return for State } n - \text{Expected Rate of Return}}{(r_n) - E(r)} \right)^2 \times \text{Probability of State } n \right]
\]

Step 3: Solve

(a) Calculating the expected return. We use the expected return equation given above to calculate the expected rate of return for the investments as follows:

\[
E(r) = r_1Pb_1 + r_2Pb_2 + \ldots + r_nPb_n
\]

\[
E(r) = (11\% \times 0.30) + (15\% \times 0.40) + (19\% \times 0.30) = 15.00\%
\]

(b) Calculating the standard deviation.
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Problem 7-7 (cont.)

Next, we calculate the standard deviation using the equation given above as follows:

\[
\sigma = \sqrt{\left( [r_1 - E(r)]^2 P_b_1 \right) + \left( [r_2 - E(r)]^2 P_b_2 \right) + \ldots + \left( [r_n - E(r)]^2 P_b_n \right)} \\
\sigma = \sqrt{\left( (11\% - 15.00\%)^2 (0.30) + (15\% - 15.00\%)^2 (0.40) + (19\% - 15.00\%)^2 (0.30) \right)} \\
\sigma = \sqrt{9.60\%} = 3.10\%
\]

**Step 4: Analyze**

**a.** The expected return for common stock A is 15.00%; and the standard deviation, which is a measure of the average or expected dispersion of the investment returns, is equal to 3.10%.

Repeat Steps 1 and 2 for Stock B.

**Step 3: Solve for stock B**

(a) Calculating the expected return. We use the expected return equation given above to calculate the expected rate of return for the investments as follows:

\[
E(r) = r_1 P_b_1 + r_2 P_b_2 + \ldots + r_n P_b_n \\
E(r) = (-5\% \times 0.20) + (6\% \times 0.30) + (14\% \times 0.30) + (22\% \times 0.20) = 9.40\%
\]

(b) Calculating the standard deviation.

Next, we calculate the standard deviation using the equation given above as follows:

\[
\sigma = \sqrt{\left( [r_1 - E(r)]^2 P_b_1 \right) + \left( [r_2 - E(r)]^2 P_b_2 \right) + \ldots + \left( [r_n - E(r)]^2 P_b_n \right)} \\
\sigma = \sqrt{\left( (-5\% - 9.40\%)^2 (0.20) + (6\% - 9.40\%)^2 (0.30) + (14\% - 9.40\%)^2 (0.30) + (22\% - 9.40\%)^2 (0.20) \right)}
\]
Problem 7-7 (cont.)

\[ \sigma = \sqrt{83.04\%} = 9.11\% \]

Step 4: Analyze

b. The expected return for common stock B is 9.40%; and the standard deviation, which is a measure of the average or expected dispersion of the investment returns, is equal to 9.11%.

c. Stock A is the better investment because it has a higher expected rate of return with less risk.
Problem 7-8

(Related to Checkpoint 7.2) (Calculating the geometric and arithmetic average rate of return) Caswell Enterprises had the following end-of-year stock prices over the last five years and paid no cash dividends:

<table>
<thead>
<tr>
<th>Time</th>
<th>Caswell</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
</tr>
<tr>
<td>2</td>
<td>$15</td>
</tr>
<tr>
<td>3</td>
<td>$12</td>
</tr>
<tr>
<td>4</td>
<td>$9</td>
</tr>
<tr>
<td>5</td>
<td>$10</td>
</tr>
</tbody>
</table>

a. Calculate the annual rate returns for each year from the above information.
b. What is the arithmetic average rate of return earned by investing in Caswell's stock over this period?
c. What is the geometric average rate of return earned by investing in Caswell's stock over this period?
d. Considering the beginning and ending stock prices for the five-year period are the same, which type of average rate of return best describes the average annual rate of return earned over the period (the arithmetic or geometric)?

Step 1: Picture the problem

Caswell's stock price at the end of each of the past five years is shown on the following graph:
Problem 7-8 (cont.)

Step 2: Decide on a solution strategy

a. First the annual rate of return for each period must be found using the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price} + \text{Cash Distribution} - \text{Beginning Price}}{\text{Beginning Price}}
\]

b. Next the arithmetic average annual rate of return must be found by summing the annual rates of return over the past four years and dividing by 4.

c. Finally the geometric average annual rate of return can be found using the following equation:
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Problem 7-8 (cont.)

Geometric Average Return = \[ \left[ \left( 1 + \text{Rate of Return for Year 1, } r_{year 1} \right) \times \left( 1 + \text{Rate of Return for Year 2, } r_{year 2} \right) \times \cdots \times \left( 1 + \text{Rate of Return for Year } n, r_{year n} \right) \right]^{\frac{1}{n}} - 1 \]

Step 3: Solve

a. Calculate the annual rate of return for each period using the rate of return equation given above:

Rate of Return, \( r \) = \( \frac{\text{Current Value} - \text{Previous Value}}{\text{Previous Value}} \)

End of time 2:

Rate of Return, \( r \) = \( \frac{15 + 0 - 10}{10} = 50.00\% \)

End of time 3:

Rate of Return, \( r \) = \( \frac{12 + 0 - 15}{15} = -20.00\% \)

End of time 4:

Rate of Return, \( r \) = \( \frac{9 + 0 - 12}{12} = -25.00\% \)

End of time 5:

Rate of Return, \( r \) = \( \frac{10 + 0 - 9}{9} = 11.11\% \)

b. Calculate the arithmetic average annual return by summing the annual rates of return and dividing by 4

Arithmetic Average Return = \( \frac{(50.00\%) + (-20.00\%) + (-25.00\%) + (11.11\%)}{4} = 4.03\% \)

c. Calculate the geometric average annual return using the equation given above:
Problem 7-8 (cont.)

Geometric Average Return

\[
\frac{1}{4} \left[ (1 + (50.00\%)) \times (1 + (-20.00\%)) \times (1 + (-25.00\%)) \times (1 + (11.11\%)) \right] - 1 = 0.00\%
\]

Step 4: Analyze

The arithmetic average annual return is 4.03%, while the geometric average annual return is 0.00%.

d. The geometric average annual return best describes the average annual rate of return. Since the starting and ending stock prices are the same, the average rate of return should be zero, and it is zero in our calculation of geometric annual rate of return. The geometric average rate of return reflects the effect of compounding and thus answers the question of what is the rate of return over a multi-year time period.
Problem 7-9

(Calculating the geometric and arithmetic average rate of return) The common stock of the Brangus Cattle Company had the following end-of-year stock prices over the last five years and paid no cash dividends:

<table>
<thead>
<tr>
<th>Time</th>
<th>Brangus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$ 15</td>
</tr>
<tr>
<td>2</td>
<td>$ 10</td>
</tr>
<tr>
<td>3</td>
<td>$ 12</td>
</tr>
<tr>
<td>4</td>
<td>$ 23</td>
</tr>
<tr>
<td>5</td>
<td>$ 25</td>
</tr>
</tbody>
</table>

a. Calculate the annual rate of return for each year from the above information.
b. What is the arithmetic average rate of return earned by investing in Brangus Cattle Company's stock over this period?
c. What is the geometric average rate of return earned by investing in Brangus Cattle Company's stock over this period?
d. Which type of average rate of return best describes the average annual rate of return earned over the period (the arithmetic or geometric)? Why?

Step 1: Picture the problem

Brangus Cattle Company's stock price at the end of each of the past five years is shown on the following graph:
Problem 7-9 (cont.)

Step 2: **Decide on a solution strategy**

a. First the annual rate of return for each period must be found using the following equation:

\[
\text{Rate of Return} = \frac{\text{Cash Return}}{\text{Beginning Price}} = \frac{\text{Ending Price}}{\text{Beginning Price}} + \frac{\text{Cash Distribution}}{\text{Beginning Price}} - \frac{\text{Beginning Price}}{\text{Beginning Price}}
\]

b. Next the arithmetic average annual rate of return must be found by summing the annual rates of return over the past four years and dividing by 4.

c. Finally the geometric average annual rate of return can be found using the following equation:
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Problem 7-9 (cont.)

Geometric Average Return = \left( \left( 1 + \frac{\text{Rate of Return for Year 1, } r_{year\, 1}}{} \right) \times \left( 1 + \frac{\text{Rate of Return for Year 2, } r_{year\, 2}}{} \right) \times \cdots \times \left( 1 + \frac{\text{Rate of Return for Year } n, \, r_{year\, n}}{} \right) \right)^{\frac{1}{n}} - 1

Step 3: Solve

a. Calculate the annual rate of return for each period using the rate of return equation given above:

\text{Rate of Return, } r_{\text{end of time 2}} = \frac{\$10 + \$0 - \$15}{\$15} = -33.33\%

\text{Rate of Return, } r_{\text{end of time 3}} = \frac{\$12 + \$0 - \$10}{\$10} = 20.00\%

\text{Rate of Return, } r_{\text{end of time 4}} = \frac{\$23 + \$0 - \$12}{\$12} = 91.67\%

\text{Rate of Return, } r_{\text{end of time 5}} = \frac{\$25 + \$0 - \$23}{\$23} = 8.70\%

b. Calculate the arithmetic average annual return by summing the annual rates of return and dividing by 4:

\text{Arithmetic Average Return} = \frac{(( -33.33\%) + (20.00\%) + (91.67\%) + (8.70\%))}{4} = 21.76\%

c. Calculate the geometric average annual return using the equation given above:
Problem 7-9 (cont.)

Geometric Average Return

\[
\frac{1}{4} \left[ (1 + (-33.33\%)) \times (1 + (20.00\%)) \times (1 + (91.67\%)) \times (1 + (8.70\%)) \right] - 1 = 13.62\%
\]

**Step 4: Analyze**

The arithmetic average annual return is 21.76%, while the geometric average annual return is 13.62%.

d. Geometric average return best describes the average annual rate of return over a period because it takes compounding into account, so it answers the question concerning the expected rate of return over a multi-year period. The arithmetic average rate of return is just a simple average, so it is useful for questions concerning the expected rate of return for a year.
Problem 7-10

(Comprehensive problem) Use the following end of year stock price data to answer the questions found below for the Barris and Carson companies.

<table>
<thead>
<tr>
<th>Time</th>
<th>Barris</th>
<th>Carson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
<td>$20</td>
</tr>
<tr>
<td>2</td>
<td>$12</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>$8</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>$15</td>
<td>27</td>
</tr>
</tbody>
</table>

a. Compute the annual rates of return for each time period and for both firms.
b. Calculate both the arithmetic and geometric mean rates of return for the entire three-year period using your annual rates of return from part a above. Note - you may assume that neither firm pays any dividends.
c. Compute a three-year rate of return spanning the entire period (i.e., using the ending price for period 1 and ending price for period 4).
d. Since the rate of return calculated in c above is a 3-year rate of return, convert it to an annual rate of return by using the following equation:

\[
\left(1 + \frac{\text{Three-Year Rate of Return}}{3}\right) = \left(1 + \frac{\text{Annual Rate of Return}}{3}\right)^3
\]
e. How is the annual rate of return calculated in part d above related to the geometric rate of return? When you are evaluating the performance of an investment that has been held for several years, what type of average rate of return should you use (arithmetic or geometric)? Why?

Find the Arithmetic average rate of return.

Step 1: Picture the problem

Barris and Carson companies' stock price at the end of the past four years is shown on the following graphs:
Step 2: Decide on a solution strategy

a. First the annual rate of return for each period must be found using the following equation:

\[
\text{Rate of Return} = \frac{\text{Ending Price} \times \text{Cash Distribution (Dividends)} - \text{Beginning Price}}{\text{Beginning Price}}
\]

b. Next the arithmetic average annual rate of return must be found by summing the annual rates of return over the past four years and dividing by 3.

Finally, the geometric average annual rate of return can be found using the following equation:

\[
\text{Geometric Average Return} = \left[ \left( 1 + \frac{\text{Rate of Return for Year 1, } r_{\text{year 1}}}{\text{Beginning Price}} \right) \times \left( 1 + \frac{\text{Rate of Return for Year 2, } r_{\text{year 2}}}{\text{Beginning Price}} \right) \times \ldots \times \left( 1 + \frac{\text{Rate of Return for Year } n, \ r_{\text{year n}}}{\text{Beginning Price}} \right) \right]^{\frac{1}{n}} - 1
\]
Step 3: Solve

a. Calculate the annual rate of return for each period using the rate of return equation given above:

\[
\text{Rate of Return, } r = \frac{\text{Ending Value} + \text{Payments} - \text{Beginning Value}}{\text{Beginning Value}} \times 100\%
\]

- **Barris:**
  - End of time 2: \( r = \frac{12 + 0 - 10}{10} = 20.00\% \)
  - End of time 3: \( r = \frac{8 + 0 - 12}{12} = -33.33\% \)
  - End of time 4: \( r = \frac{15 + 0 - 8}{8} = 87.50\% \)

- **Carson:**
  - End of time 2: \( r = \frac{28 + 0 - 20}{20} = 40.00\% \)
  - End of time 3: \( r = \frac{32 + 0 - 28}{28} = 14.29\% \)
  - End of time 4: \( r = \frac{27 + 0 - 32}{32} = -15.63\% \)

b. Calculate the arithmetic average annual return by summing the annual rates of return and dividing by 3:

\[
\text{Arithmetic Average Return} = \frac{20.00\% + (-33.33\%) + 87.50\%}{3} = 24.72\%
\]

- **Barris:**
  - \( \text{Arithmetic Average Return} = 24.72\% \)

- **Carson:**
  - \( \text{Arithmetic Average Return} = 12.89\% \)

Calculate the geometric average annual return using the equation given above:

\[
\text{Geometric Average Return} = \left[ (1 + \text{Rate of Return}) \times (1 + \text{Rate of Return}) \times (1 + \text{Rate of Return}) \right]^{\frac{1}{\text{Number of Years}}} - 1
\]

- **Barris:**
  - \( \text{Geometric Average Return} = \left[ 1 + (20.00\%) \times (1 + (-33.33\%)) \times (1 + (87.50\%)) \right]^{\frac{1}{3}} - 1 = 14.47\% \)

- **Carson:**
  - \( \text{Geometric Average Return} = \left[ 1 + (40.00\%) \times (1 + (14.29\%)) \times (1 + (-15.63\%)) \right]^{\frac{1}{3}} - 1 = 10.52\% \)
Problem 7-10 (cont.)

**Step 4: Analyze**

**a.** The annual rate returns for each year are shown in the table below.

<table>
<thead>
<tr>
<th>Time</th>
<th>Value of Barris Stock</th>
<th>Annual Rate of Return</th>
<th>Value of Carson Stock</th>
<th>Annual Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
<td>20.00%</td>
<td>$20</td>
<td>40.00%</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>–33.33%</td>
<td>28</td>
<td>14.29%</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>87.50%</td>
<td>32</td>
<td>15.63%</td>
</tr>
</tbody>
</table>

**b.** The arithmetic average rate of return earned by investing in Barris stock over this period is 24.72% and the arithmetic average rate of return earned by investing in Carson stock over this period is 12.89%.

The geometric average rate of return earned by investing in Barris stock over this period is 14.47% and the geometric average rate of return earned by investing in Carson stock over this period is 10.52%.

**Using a holding period rate of return to find an arithmetic average annual rate of return and a geometric average annual rate of return.**

**Step 1: Picture the problem**

The rate of return you would have earned over a 3-year period investing in Barris and in Carson stock is determined by the price paid at the beginning of the period, the price received when the shares are sold at the end of the year, and the cash dividend.
Step 2: Decide on a solution strategy

The rate of return earned by holding each stock for the three-year period is calculated by the following equation:

\[
\text{Three-Year Rate of Return} = \frac{\text{Ending Price, Time 4} + \text{Cash Distribution, Time 4} - \text{Beginning Price, Time 1}}{\text{Beginning Price, Time 1}}
\]

Then to convert the three-year return to the annual return use the following equation solving for the annual rate of return:

\[
\left(1 + \frac{\text{Three-Year Rate of Return}}{\text{Rate of Return}}\right) = \left(1 + \frac{\text{Annual Rate of Return}}}{\text{Rate of Return}}\right)^3
\]

Step 3: Solve

Substituting into the three-year rate of return equation we get the following results for Barris and Carson:
Problem 7-10 (cont.)

Barris: Three-Year Rate of Return

\[
\begin{align*}
\text{Ending Price, Time 4} & \quad + \quad \text{Cash Distribution, Time 1} \quad - \quad \text{Beginning Price, Time 1} \\
\text{Beginning Price, Time 1} & \quad = \quad \frac{\$15 + \$0 - \$10}{\$10} \quad = \quad 50.00\% \\
\end{align*}
\]

Carson: Three-Year Rate of Return

\[
\begin{align*}
\text{Ending Price, Time 4} & \quad + \quad \text{Cash Distribution, Time 1} \quad - \quad \text{Beginning Price, Time 1} \\
\text{Beginning Price, Time 1} & \quad = \quad \frac{\$27 + \$0 - \$20}{\$20} \quad = \quad 35.00\% \\
\end{align*}
\]

Then substituting into the conversion equation we get the following results for Barris and Carson:

\[
\begin{align*}
\text{Barris:} \quad (1 + 50.00\%) & = \left(1 + \frac{\text{Annual Rate of Return}}{3}\right)^3 \\
\text{Carson:} \quad (1 + 35.00\%) & = \left(1 + \frac{\text{Annual Rate of Return}}{3}\right)^3 \\
\end{align*}
\]

\[
\begin{align*}
\text{Annual Rate of Return} \quad & = \left(1 + 50.00\%\right)^{\frac{1}{3}} - 1 \\
\text{Annual Rate of Return} \quad & = \left(1 + 35.00\%\right)^{\frac{1}{3}} - 1 \\
\end{align*}
\]

\[
\begin{align*}
\text{Annual Rate of Return} \quad & = 14.47\% \\
\text{Annual Rate of Return} \quad & = 10.52\% \\
\end{align*}
\]

Step 4: Analyze

c. The three-year rate of return spanning the entire period for Barris is 50.00% and the three-year rate of return spanning the entire period for Carson is 35.00%.

d. The annual rate of return converted from the three-year rate in part c for Barris is 14.47% and the annual rate of return converted from the three-year rate in
Chapter 7

Problem 7-10 (cont.)

part c for Carson is 10.52%.

e. The annual rate of return calculated in part d is nothing but geometric average rate of return calculated in a different way.

The geometric rate of return should be used when you are evaluating the performance of an investment that has been held for several years. It best describes the average annual rate of return over a multi-year period because it compounds at the same rate while the arithmetic rate of return is best used for a single period.